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Handling synergies in Multiple Criteria Behavioural Group Decision Aiding problem for designing post-industrial cultural tourism product

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Outline

- Cultural tourism products (CTPs) and their role for LGUs
- The CTP design problem
- Synergies and their inclusion in CTP design problem
- Designing the optimal post-industrial CTP for Czeladź Commune (Poland)
- Conclusions and future research

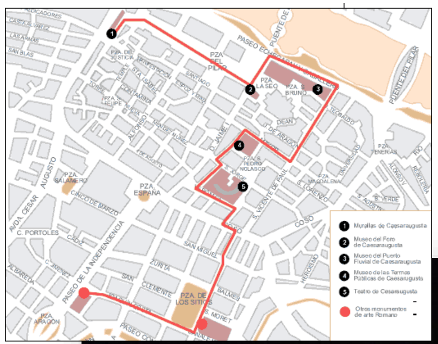
Motivation

Cultural tourism products and their role

- Developing cultural tourism helps Local Goverment Units (LGUs) **attract tourist flows and investment** to the area they manage (Kay Smith *et al*. 2022, Bec *et al*. 2021)

- Cultural Tourism Product (CTP) is **anything that can be offered to tourists** for participating in cultural tourism **to satisfy their cultural needs and wants** by using the cultural tourism resource as a basis (Chiriko 2020, Copley and Ian 1996)

- CTPs are usually **complex**, consisting of several linked mono-products like heritage sites, events, trails, gastronomy, or crafts (Żabiński 2012, Levitt 2008)



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Czeladź Commune and its post-industrial heritage

- Czeladź is a small town (over 30k inhabitants) in the northern part of the Metropolis GZM area in **Southern Poland**, the **oldest town** (founded in 13th century) in Będzin County

- Since the 1860s, the town began to **turn into an industrial centre** – **two coal mines** were established, leaving behind a rich and diverse heritage, including:
  - historic post-industrial buildings and machinery
  - houses for white-collar workers, skilled workers, officials
  - villas of coal mines directors
  - mine administration buildings
  - social, culture, school buildings
  - Neo-Romanesque parish church




How to design CTP for a Post-Industrial Town?

Multiple Criteria Behavioural Group Decision Aiding problem

Key Challenges and Considerations	Required approaches
The diversity of **CTP formats** and post-industrial **heritage assets** (Mason 2016, Stokes 2008, Yu and Xu 2019)	creating complex CTP – building **portfolio** of simple mono-products
Limited financial **resources** (Lee-Ross and Pryce 2010, Nogueira and Pinho 2015)	**mathematical programming** for solving knapsack problem
CTP variability in **attributes** tailored to different **objectives** (Lohmann 2004, Mason 2016)	use of **MCDA methods** to compare alternative solutions
Many **DMs and stakeholders** involved with (Russo and van der Borg 2002, Vucetic 2009): ○ various **goals** and **priorities** ○ **cognitive limitations** that can affect the efficient use of MCDA methods	○ use of **GDM methods** to aggregate stakeholders' opinions ○ use of **behavioural approach** to identify cognitive capabilities

CTP design problem

Formalization

$$\mathcal{P} = \{C, A, I, \mathbb{F}, P, \mathbb{S}, S^{G}, E, R^{S}, R^{E}, \mathrm{CCTP}\}, \quad (1)$$

where:

C – set of CTP component types, $C = \{C_j\}_{j=1,\dots,J}$

A – set of alternatives defined within subsets for each CTP component seperately, $A = \{a_i\}_{i=1,\dots \sum_{j=1}^{J}|A^j|} = \bigcup_{j=1}^{J} A^j$, $A^j=\{a_c^j\}_{c=1,\dots,|A^j|}$

I – set of stakeholders, $I = \{I^k\}_{k=1,\dots,K}$

$\mathbb{F}$ – set of collections of criteria used by the stakeholders to evaluate alternatives, $\mathbb{F} = \{F^k\}_{k=1,\dots,K}$, $F^k=\{f_p^k\}_{p=1,\dots,P^k}$

P – cognitive profiles characterizing given stakeholders, $P = \{P^k\}_{k=1,\dots,K}$

$\mathbb{S}$ – set of collections of stakeholders' scoring systems with cardinal scores for all alternatives from A, $\mathbb{S} = \{S^k\}_{k=1,\dots,K}$, $S^k = \{v_i^k\}_{i=1,\dots,I}$

S^{G} – set of consensual scores which represent group preferences, $S^G=\{\ddot{v}_i\}_{i=1,\dots,I}$

E – set of costs of producing alternatives from A, $E = \{e_i\}_{i=1,\dots,I}$

R^S – set of synergistic relations describing an increase of the attractiveness for given mixes of alternatives from A,

R^E – sets of synergistic relations describing cost reductions for given mixes of alternatives from A,

CCTP – mathematical program for finding optimal complex CTP.

CTP design problem

Issues in modelling synergies

- Usually, the contribution of alternative a_i to the **attractiveness synergy** within the complex CTP is defined as (Almeida and Duarte, 2011):

$$R_i^S = \frac{1}{2}\sum_{\substack{o=1 \\ o\neq i}}^{I} y_o r_{io}[\ddot{v}_i + \ddot{v}_o] + \frac{1}{3}\sum_{\substack{o=1 \\ o\neq i}}^{I-1} \sum_{p=o}^{I} y_o y_p r_{iop}\left[\ddot{v}_i + \ddot{v}_o + \ddot{v}_p\right] + \cdots, \qquad (4)$$

where:

y_o, y_p – binary variables indicating whether alternatives a_o and a_p are included in complex CTP

$r_{[list]}$ – percentage effect of attractiveness synergy for alternatives from [list] participating in the synergy

- A similar formulation may be used for costs synergy

CTP design problem

Issues in modelling synergies

- Problems with „**concise" definition** of synergies by stakeholders in CTP problem:
  - defined only by **some** stakeholders (due to cognitive requirements and expertise)
  - differences in assignments of the **strength** of synergy ($r^{k}_{[list]}$)
  - differences in identifying the **mixes of alternatives** that participate in a synergy

	I^1	I^2	I^3	I^4	I^5	I^6	I^7
a_1	30.00%				15.00%	50.00%	50.00%
a_2	30.00%				15.00%	50.00%	50.00%
a_3							
a_4	30.00%					30.00%	
a_5			25.00%	50.00%			
a_6	40.00%			50.00%			
a_7	40.00%		25.00%	50.00%			
a_8	40.00%						
a_9						30.00%	
a_{10}			25.00%			50.00%	
a_{11}			25.00%				

CTP design problem

Modelling synergies consensually

- To address the challenges posed by individual synergy declarations, our approach proposes:

  1. Designing mechanisms to combine synergy strength for **overlapping synergies** (those that share common subsets of alternatives) by **defining synergy clusters**
  2. Introducing mechanisms to **prevent double counting of synergy strengths,** ensuring that **overlapping synergies** within clusters cannot both contribute simultaneously
  3. Modelling synergies strengths within clusters as **interval numbers** derived from diverse declarations of individual stockholders
  4. Analysing the CCTP problem via **simulation**, integrating random draws from these interval-based synergy effects into the optimization.

Incorporating synergies into CCTP

1. Synergy clusters

- We are given a set of all declared synergies:

$$R = \{R_1, R_2, \dots, R_W\}, R_w \subseteq \{1, \dots, n\},\ |R_w| \geq 2,$$

  where each R_w is a subset of indices i corresponding to alternatives a_i

- A **cluster** $\mathbb{C}^l \subseteq R$ is defined as a collection of “connected synergies”, which means any two synergies sharing at least two indices end up in the same $\mathbb{C}^l$

- To **build** a cluster:
  - We pick any unassigned synergy R_a, create a new cluster $\mathbb{C}^l = \{R_a\}$, and mark R_a as assigned
  - For each unassigned synergy R_b, we check whether there exists at least one $T \in \mathbb{C}^l$ such that $|R_b \cap T| \geq 2$, and if so, we add it into cluster $\mathbb{C}^l$

CTP design problem
2. Synergy supersets

- To **avoid double counting** of synergies within clusters:
  - we form **new superset synergies** $R_u = R_a \cup R_b$ for each pair pair $(R_a, R_b) \subset \mathbb{C}^l$ that satisfies

$$R_a \not\subset R_b, R_b \not\subset R_a \wedge |R_a \cap R_b| \geq 2 \text{ (neither is strictly contained in the other)},$$

  - The procedure **iteratively includes all synergies and superset synergies** that may co-occur within each cluster
- For each cluster $\mathbb{C}^l$ we introduce a **binary control variable** for each original synergy and their derived supersets $R_a \in \mathbb{C}^l$

$$y_a^l \in \{0; 1\}, y_a^l = 1 \leftrightarrow (\forall\, i \in R_a : \; x_i = 1)$$

where x_i indicates whether alternative a_i is included in CTP

CTP design problem

3. Constraints for exclusion of overlapping synergies in clusters

- For each cluster $\mathbb{C}^l$ we identify **strict subsets of synergies**, which we write

$$R_a \subsetneq R_b \leftrightarrow R_a \subseteq R_b \land |R_a| < |R_b|$$

- We formulate **exclusion constraints within clusters, for all strictly contained pairs**, i.e., $(R_a, R_b) \in \left(\mathbb{C}^l \cup \{\text{supersets}\}\right)^2$ with $R_a \subsetneq R_b$

$$y_a^l + y_b^l \leq 1.$$

CTP design problem

4. Representation of synergies strengths (1)

- Stakeholders declare **percent gain** for synergies they identified, e.g. I^k declares $r_{ak} \in [0,1]$ for R_a
- We define consensual synergies depending on the situation:

1. For a given **base synergy** R_a that is not a part of any other overlapping synergy in the cluster ($\neg\exists T \in \mathbb{C}^l$ such that $R_a \subsetneq T$)

$$r_a^l = \left[\min_k r_{ak}, \max_k r_{ak}\right] = [\mathrm{LB}(R_a), \mathrm{UB}(R_a)]$$

and the bonus for CTP score for this synergy is calculated as

$$y_a^l \left(\sum_{i \in R_a} \ddot{v}_i\right) r_a^l$$

CTP design problem

4. Representation of synergies strengths (2)

2. **Overlapping synergies**

- For declared synergy R_b that overlaps other synergy R_a $(R_a \subseteq R_b)$, R_b is split into
  - Common part $\boldsymbol{C} = \boldsymbol{R_a}$ with $r_C^l = [\max(\mathrm{LB}(R_a), \mathrm{LB}(R_b)), \max(\mathrm{UB}(R_a), \mathrm{UB}(R_b))]$
  - Exclusive part $\boldsymbol{E} = \boldsymbol{R_b} \setminus \boldsymbol{R_a}$ with $r_E^l = r_b^l$

  and the bonus for CTP score for this synergy is calculated as

$$y_b^l[(\sum_{i \in C} \ddot{v}_i) r_C^l + (\sum_{j \in E} \ddot{v}_j) r_E^l]$$

- Similar rule applies for the overlaps with more than one synergy

CTP design problem

4. Representation of synergies strengths (3)

3. **Supersets of synergies**

   - For declared synergy $R_u = R_a \cup R_b$ where neither R_a nor R_b overlaps fully each other we have:

     - Common part $\boldsymbol{C} = \boldsymbol{R_a} \cap \boldsymbol{R_b}$ with $r_C^l = [\max(\mathrm{LB}(R_a), \mathrm{LB}(R_b)), \max(\mathrm{UB}(R_a), \mathrm{UB}(R_b))]$
     - Exclusive parts: (1) $\boldsymbol{E_a} = \boldsymbol{R_a} \setminus \boldsymbol{C}$ with $r_{E_a}^l = r_a^l$ and (2) $\boldsymbol{E_b} = \boldsymbol{R_b} \setminus \boldsymbol{C}$ with $r_{E_b}^l = r_b^l$

   and the bonus for CTP score for this synergy is calculated as

$$y_u^l \left[\left(\sum_{i \in C} \ddot{v}_i \right) r_c^l + \left(\sum_{j \in E_a} \ddot{v}_j \right) r_{E_a}^l + \left(\sum_{j \in E_b} \ddot{v}_j \right) r_{E_b}^l \right]$$

CTP design problem

Example of synergies formalization

- Stakeholders identify the following synergies and their strengths →
- Cluster $\mathbb{C}^1$ is defined as $\{R_1 = \{1,2\}, R_2 = \{1,2,4\}, R_3 = \{1,2,10\}\}$
- A superset synergy is built out of R_2 and R_3 determined as $R_4 = \{1,2,4,10\}$
- We have four binary control variables: $y_1^1 + \ y_2^1, +y_3^1 + y_4^1 \leq 1$
- We have the following synergy strengths:
  - $r_1^1 = [0.15;\ 0.50]$,
  - $r_2^1 = \{r_{C_2}^1 = [0.30; 0.50], r_{E_2}^1 = [0.30; 0.30]\}$,
  - $r_3^1 = \{r_{C_3}^1 = [0.40; 0.50], r_{E_3}^1 = [0.40; 0.40]\}$,
  - $r_4^1 = \left\{r_{C_4}^1 = [0.40; 0.50], r_{E_4^1}^1 = [0.30; 0.30], r_{E_4^2}^1 = [0.40; 0.40], \right\}$

	I^1	I^3	I^4	I^5	I^6	I^7
a_1	30%			15%	40%	50%
a_2	30%			15%	40%	50%
a_3						
a_4	30%				30%	
a_5		25%	50%			
a_6	40%		50%			
a_7	40%	25%	50%			
a_8	40%					
a_9					30%	
a_{10}		25%			40%	
a_{11}		25%				

CTTP problem

Building optimization model

$$
CCTP \begin{cases} V = \sum_{i=1}^{|A|} x_i \ddot{v}_i + \sum_{l=1}^{|\mathbb{C}|} \sum_{m=1}^{|\mathbb{C}^l|} y_m^l \hat{r}_m^l \sum_{\substack{i \in R_m \\ R_m \subset \mathbb{C}^l}} \ddot{v}_i \rightarrow max \\ \text{s.t.} \\ \quad \sum_{i=1}^{|A|} x_i e_i \leq W \\ \quad y_m^l \leq x_i, \forall R_m \in \mathbb{C}^l, \forall i \in R_m \\ \quad y_m^l + y_n^l \leq 1, \ \forall \mathbb{C}^l \in \mathbb{C}, \forall R_m, R_n \in \mathbb{C}^l, \ \forall m < n \\ \quad CC \end{cases}
$$

where:

$\mathbb{C}$ – set of all synergy clusters,

W – budget for CTP,

CC – complementarity constraints (e.g. exclusion of alternatives from the same category, kind or type, or variants that contain other simpler CTP alternatives within themselves).

$\hat{r}_m^l$ - an instance of synergy strength randomly picked up from r_m^l

Designing the best post-industrial CTP for Czeladź

Structuring the CTP problem (A)

Component	Alternative	Details	Component	Alternative	Details
C_1	a_1	Map	C_3	a_{12}	Post-industrial picnic
	a_2	Guidebook		a_{13}	Czeladź Post-Industrial Festival
	a_3	Board game – "Industrialist"		a_{14}	Mobile post-industrial exhibitions
	a_4	Educational tour		a_{15}	Post-industrial concert and music workshops
	a_5	Set of promotional gadgets		a_{16}	Czeladź Post-Industrial Rally
C_2	a_6	Guidebook on the web		a_{17}	Post-industrial lights and lasers
	a_7	Multimedia guidebook-gadget	C_4	a_{18}	Mix trail
	a_8	Virtual tour on the web		a_{19}	Questing of "Saturn" mine heritage
	a_9	Augmented "virtuality"		a_{20}	Questing of "Czeladź" mine heritage
	a_{10}	Questing on the web		a_{21}	Mining industry trail
	a_{11}	Trail on the web		a_{22}	Real existing post-industrial heritage trail

Designing the best post-industrial CTP for Czeladź

Structuring the CTP problem (*I*)

- **DM** is a **mayor** and the **commune management board**, and they seek **opinions** regarding the attractiveness of various alternatives of CTPs from **21 potential stakeholders**:

No. of stakeholders	Place of work
2	Saturn Museum in Czeladź in the department: **The "Elektrownia" Contemporary Art Gallery (CAG)**
3	**management staff** of the **Saturn Museum** and the department: **The "Elektrownia" CAG**
1	**management staff** of the Department of City Promotion, Culture and International Cooperation of the **Czeladź City Hall**
1	**Department of City Promotion, Culture and International Cooperation** of the Czeladź City Hall
1	**management staff** of the **Municipal Sport and Recreation Centre** in Czeladź
6	**headmasters and deputy headmasters** of educational institutions in Czeladź
7	teaching **staff of educational institutions** in Czeladź

Designing the best post-industrial CTP for Czeladź

Aggregating results and determining group scores (S^G)

- Minimum cost consensus based on the **Ben-Arieh's approach** was used for preference **aggregation**

Alternative (a^i)	Individual evaluations (v_i^k)				Consensus scores ($\ddot{v}_i$)
	Stakeholder 1 (from TOPSIS)	Stakeholder 2 (from SMART)	…	Stakeholder 22 (from AHP)	
a1 – Map	0.012	0.067	…	0.180	0.151
a2 – Guidebook	0.030	0.158	…	0.226	0.269
a3 – Board game – "Industrialist"	0.059	0.016	…	0.124	0.151
a4 – Educational Tour	0.010	0.107	…	0.149	0.244
…	…	…	…	…	…
a16 – Czeladź Post-Industrial Rally	0.029	0.017	…	0.113	0.141
a17 – Post-industrial lights and lasers	0.093	0.128	…	0.173	0.210
a18 – Mix Trail	0.372	0.253	…	0.194	0.374
a19 – Questing of "Saturn" mine heritage	0.140	0	…	0	0.259
a20 – Questing of "Czeladź" mine heritage	0.046	0	…	0.025	0.199
a21 – Mining industry trail	0.071	0.157	…	0.157	0.131
a22 – Real existing post-industrial heritage trail	0.145	0.161	…	0.150	0.275
Weights	**0.13**	**0.13**	…	**0.07**	-

Designing the best post-industrial CTP for Czeladź

Defining synergies

	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10	R11	R12	R13	R14	R15	R16	R17	R18	R19	R20	R21	R22	R23	R24	R25	R26	R27
a1	0.3										0.5		0.5						0.6			0.6		0.15			
a2	0.3										0.5		0.5						0.6			0.6		0.15			
a3														0.25			0.1	0.5							0.15		
a4	0.3											0.3															
a5																									0.15		
a6				0.4																							
a7																											
a8				0.4	0.25		0.7											0.5								0.15	
a9		0.4					0.7	0.5																		0.15	
a10		0.4			0.25		0.7		0.3																		
a11		0.4																									
a12						0.25				0.6		0.3			0.3												
a13						0.25			0.3							0.25											0.15
a14								0.5									0.1										0.15
a15						0.25									0.3					0.4			0.4				
a16						0.25										0.25											
a17						0.25														0.4			0.4				
a18											0.5																
a19			0.25		0.25					0.6				0.25							0.4						
a20			0.25		0.25					0.6											0.4						
a21																											
a22												0.3															
	(01) Marek			(03) Jakub	(05) Marta		(06) Agnieszka	(07) Artur		(10) Ewelina	(14) Mateusz		(15) Lucyna					(19) Jakub	(21) Aneta			(22) Anna		Stefania (24)			

Designing the best post-industrial CTP for Czeladź

Clusters identified and supersets

Cluster	Alternatives in synergy	Originally defined by	Label
1	a1,a2,a18	R11	S1
	a1,a2,a4	R1	S2
	a1,a2	R13,R19,R22,R24	S3
2	a8,a10,a19,a20	R5	S4
	a12,a19,a20	R10	S5
	a8,a9,a10	R7	S6
	a9,a10,a11	R2	S7
	a19,a20	R3,R21	S8
	a8,a9	R26	S9
3	a6,a8	R4	S10
4	a12,a13,a15,a16,a17	R6	S11
	a12,a15	R15	S12
	a13,a16	R16	S13
	a15,a17	R20,R23	S14
5	a9,a14	R8	S15
6	a10,a13	R9	S16
7	a4,a12,a22	R12	S17
8	a3,a19	R14	S18
9	a3,a14	R17	S19
10	a3,a8	R18	S20
11	a3,a5	R25	S21
12	a13,a14	R27	S22

Cluster	Supersets of synergy	Originally defined by	Label
1	a1,a2,a4,a18	S1,S2	S23
2	a8,a10,a12,a19,a20	S4,S5	S24
	a8,a9,a10,a19,a20	S4,S6	S25
	a8,a9,a10,a11	S6,S7	S26
	a8,a9,a10,a11,a19,a20	S4,S26	S27
	a8,a9,a10,a12,a19,a20	S5,S25	S28
	a8,a9,a10,a11,a12,a19,a20	S24,S26	S29

Designing the best post-industrial CTP for Czeladź

Constraints and synergy strengths

- We solve the CCTP problem with a **budget** not exceeding **90,000 PLN** and three **additional constraints** defined by the DM, requiring:
  - At least one questing to be included in the CTP
  - Only one real trail can be built
  - As the "mixed trail" includes the online trail, both cannot be included in the CTP:

- Based on the stakeholders' **synergy strength declarations**, random parameters were used:
  - `obj[y1]  <- runif(1,0.5,0.6) * (v[1] + v[2]) + 0.5 * v[18]`
  - `obj[y2]  <- runif(1,0.3,0.6) * (v[1] + v[2]) + 0.3 * v[4]`
  - `obj[y3]  <- runif(1,0.15,0.6) * (v[1] + v[2])`
  - `obj[y4]  <- 0.25 * (v[8] + v[10]) + runif(1,0.25,0.4)*(v[19] + v[20])`
  - `obj[y8]  <- runif(1,0.25,0.4) * (v[19] + v[20])`
  - `obj[y23] <- runif(1, 0.5, 0.6) * (v[1] + v[2]) + 0.3 * v[4] + 0.5 * v[18]`
  - `obj[y25] <- 0.7 * (v[8] + v[9] + v[10]) + runif(1, 0.25, 0.4) * (v[19] + v[20])`

Designing the best post-industrial CTP for Czeladź

Solution

- To find the general CTP recommendation we used **Monte Carlo simulation with 5000 replications**
- For all replications, **the same solution** was recommended with the following mono products included:
  - a1 – Map
  - a2 – Guidebook
  - a4 – Educational Tour
  - a6 – Guidebook on the web
  - a7 – Multimedia guidebook-gadget
  - a15 – Post-industrial concert and music workshops
  - a17 – Post-industrial lights and lasers
  - a18 – Mix trail
  - a19 – Questing of ”Saturn” mine heritage
  - a20 – Questing of ”Czeladź” mine heritage
  - a22 – Real existing post-industrial heritage trail

Synergy S14 (a15, a17)

Synergy S8 (a19, a20)

Superset S23 = { S1 ∪ S2 ∪ S3 } (a1 – a18)

Conclusions

- We developed a theoretical model of the CTP design process that incorporates multi-criteria, group, and behavioural aspects, including an **effective group synergy handling mechanism:**
  - Synergies are defined by stakeholders, and **consensual strengths** are modelled as **interval numbers**
  - A mechanism to **avoid double counting** of overlapping synergies is introduced, which requires identifying **synergy clusters** and determining **supersets of synergies** within clusters
- Similarly, **cost synergies** that reduce the cost of selected CTP mono-product bundles **may be incorporated** into the CCTP model
- The **imprecision of synergistic effects** can be included in the model by assuming that each stakeholder's declaration is itself an interval number affected by **some error in its lower and upper bounds**

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